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METHOD OF PHYSICAL MODELING AND ITS POTENTIALITIES

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Based on the theory of modeling with the use of several models, it has been shown that all the known methods of investigation of this type satisfy the same requirements. The possibility of determining the sought results characterizing one phenomenon or another from the measured quantities of a phenomenon that has no common parameters with the phenomenon being investigated has been substantiated. Examples of the practical use of modeling are given.

Introduction. The intense development of different technologies in industry and in branches servicing it determined the necessity of using widely physical methods of investigation for their needs. One such important method is modeling (method of models), which can be used not only for proper understanding of one process or another but also for monitoring and controlling it [1, 2].

The requirement (similarity principle) [1] that must be satisfied by the object of investigation (model) reproducing or describing mathematically one phenomenon or another has been formulated based on ideas of the methods of investigation of physical phenomena, which existed in the early twentieth century. The idea that modeling as a process of investigation must be realized with a single model not only hindered further development of its theory and creation of new methods, but also led to the erroneous conclusion that a number of developed special methods that allow one to conduct investigations with a high accuracy do not satisfy the similarity principle. Thus, as far as the first statement is concerned, it is pertinent to note that, in some works, modeling implied only conducting experiments with models of smaller or larger scale as compared to a full-scale object. This is shown, for example, in [3] and does not comply with the actual potentialities of this method [4, 5, and others]. The indicated definition of modeling has been formulated based on the well-established ideas of similarity theory, according to which all the quantitative characteristics of one phenomenon can be obtained by proportional transformation from similar characteristics of another phenomenon [6]. Such an understanding of similarity is not correct with respect to the requirement formulated in [1] and, in a number of cases, it is in contradiction with the physical essence of phenomena, since the scale factor influences the laws determining the phenomenon. We note that a solution different from the methods specified by the definition of modeling [3] was proposed for the first time in the method of determination of deformations of workpieces [7], which assumed an increase in the linear dimensions of the model and a decrease in the temperature difference at its different points. The second statement is based on the generally accepted division of the method of investigation under consideration into physical modeling and analog modeling [8] which, as will be shown below, satisfies the same requirements; therefore, there is no need to make the indicated division.

We draw on the definition of the method of investigation under consideration which is presented in [9] and refine it in view of the developed theory of modeling with the use of several models [5, 10, and others] in the following manner: *modeling is a method of experimental investigation based on replacement of*

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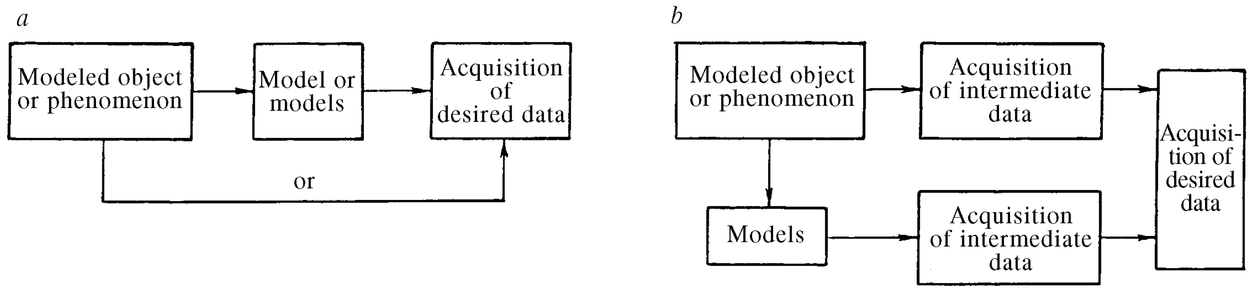

Fig. 1. Possible schemes of realization of physical modeling.

a concrete experimental object (sample) by another object or other objects similar to it (by a model or models). It is pertinent to note that in the above definition of modeling, *physically* similar objects (systems) imply objects connected to each other in such a manner that the arguments of the unknown function are the same in both objects [1]. To observe correctly the similarity conditions in passage to models, by arguments one should mean all the parameters characterizing the full-scale object, since the constant quantities characteristic of the object can change in passage to models. Failure to fulfill the similarity conditions, which, for example, can be due to a change in the geometric dimensions of objects in investigations with a single model, resulted every so often in erroneous results [5].

Extending the potentialities of the modeling realized according to the scheme of Fig. 1a and providing the use of objects suitable for investigation due to the employment of several models [4, 5, and others] is not the only feature of this method. Using it, we prove the practicability of the similarity principle also for special methods of investigation [11, 12, and others], which are realized according to a scheme different from the one above (Fig. 1b). Moreover, we substantiate the possibility of determining the characteristics of some phenomena from the characteristics of other phenomena, which have no common parameters and are not characterized by the same laws of change of the functions under study.

Experiments, Results, and Discussion. We consider an example of realization of modeling according to the scheme of Fig. 1b. It is commonly known that a number of physical phenomena are accompanied by different effects. For example, plastic deformation by tension of a ferromagnetic steel plate causes both its heating and a change in the mechanical properties of the metal [13, 14]. If this plate is locally magnetized, the value of the remanence will also change on tension [15, 16]. In this case, magnetization influences neither the mechanical nor the heat changes in the states observed as a result of the indicated deformation. Based on this fact, we can obtain results characterizing the effects observed using models reproducing the object or the phenomenon under study. In this case, one group of results obtained from the observed effects of change in the mechanical properties and the remanence will be determined for the sought properties of the object modeled (we denote them by a), while a group that is the most convenient for measurements on the object and models under study (we denote them by b) will be chosen as the second one. We compare the data of one group of measurements to the data of the other group in such a way that each pair of compared values corresponds to the same parameters of the object or phenomenon under study. Consequently, we can find a certain dependence

$$a = f(b) . \tag{1}$$

Let this dependence have the form presented in Fig. 2 (the scales of the quantities plotted on the axes are the same, i.e., the scale of the unit of the quantity a coincides with the unit of the quantity b). Having determined dependence (1), we can pass to the modeling of an object or a phenomenon of interest to us, investigating in this process the effect that makes it possible to find the values of b.

If we assume that the dependence presented in Fig. 2 as a curve exists in reality, we find a portion on it which represents a straight line positioned at an angle of 45° to the chosen axes. We can argue that on

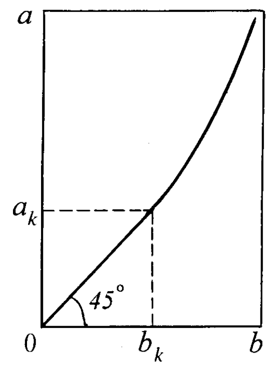

Fig. 2. Graph of a versus b; a and b are the compared quantities characterizing different effects of the modeled object or phenomenon.

this portion of the curve the laws of change of the quantities a and b in the considered effects determined by the parameters of the object or the phenomenon under study are the same. In this case, these laws of change of the quantities a and b can formally be considered as a parameter introduced with the aim of characterizing the modeled objects or phenomena and the models under consideration. As an example confirming the actual existence of the indicated feature, we refer to the fact that the value of the stress changes as a function of the deformation (Hooke law) and the relative value of the remanence changes as a function of the stress in the same manner by linear laws in tension of a steel plate in the elastic region [13, 15]. It should be noted that if the physical phenomena under study are sufficiently clearly understood, the number of measurements on models can be reduced for the ranges of variation of the quantities a and b in which they are determined by the same laws (in Fig. 2, these ranges are $[0; a_k]$ and $[0; b_k]$).

For the modeling realized according to the scheme of Fig. 1b, the relation for determining the quantity characterizing the property sought should be written in the form

$$A_{\mathrm{r}} = f\,(B_{\mathrm{r}} = b)\,, \tag{2}$$

where B_{r} is the quantity determined on a full-scale object or a phenomenon from the effect accompanying a change in the quantity A_{r}, which has one or several parameters identical to the parameters characterizing the quantity A_{r}, while the function $f(b)$ is consistent with the above procedure with the use of models. In this case, the above condition of similarity will be fulfilled in full measure, since function (1) was determined experimentally using a model similar to the full-scale object.

In all probability, relation (2) should be considered as a relation found on the basis of heuristic conjectures. However, it can be obtained both in the case of using the ideas of the structure of functional relations between physical quantities [1, 3] and when the modeling theorem is employed [5].

We prove the indicated statements, keeping, in so doing, the notation used above. The quantities a and b in equality (1) represent functions of the parameters characterizing the phenomenon. We denote these parameters for the quantities a and b by $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_q$, respectively. Then we can write

$$a = f_1\,(a_1, a_2, ..., a_{k-1}, a_k, a_{k+1}, ..., a_n) \tag{3}$$

and

$$b = f_2\,(b_1, b_2, ..., b_{k-1}, b_k, b_{k+1}, ..., b_q)\,. \tag{4}$$

It is clear that the phenomena under consideration will be characterized by certain identical parameters (material and stress). Let a_k and b_k be identical common parameters in (3) and (4). Since b, just as any function, can be represented in the form [1, 3, and others]

$$b = c' \, b_1^{m_1} \, b_2^{m_2} \, ... \, b_q^{m_q} \, , \tag{5}$$

its parameter b_k can be determined as follows:

$$b_k = f_3 \, (b) \, . \tag{6}$$

If, in (3), in place of a_k we substitute its value b_k found using another phenomenon, we obtain (1), which proves the statement. Passing to measurements on a full-scale object, one must employ (2). In such a modeling, at least three objects are involved in investigations, which corresponds to the above statement that several models must be used in the case of employment of special methods.

An analogous result can also be obtained using the theory of modeling based on several models [5] which holds that *in the case of physical modeling of an object or a phenomenon based on* $2n + 1$ *models (*$n = 0, 1, 2, ...$*), the quantity determining the sought property of the modeled object or phenomenon is found from the relation*

$$A_{\mathrm{r}} = A_{1\mathrm{m}} \frac{A_{2\mathrm{m}(0)}}{A_{3\mathrm{m}(0)}} \frac{A_{2\mathrm{m}(1)}}{A_{3\mathrm{m}(1)}} ... \frac{A_{2\mathrm{m}(n)}}{A_{3\mathrm{m}(n)}} \, , \tag{7}$$

where the quantities characterizing the sought property and entering into A_i ($i = r$, 1m, 2m(0), ..., 2m(n), 3m(n)) *as parameters must be exhausted with a partial or complete coincidence in the ratios*

$$A_{\mathrm{r}}/A_{1\mathrm{m}} \, ; \;\; A_{\mathrm{r}}/A_{2\mathrm{m}(0)} \, ; \;\; A_{1\mathrm{m}}/A_{3\mathrm{m}(0)} \, ; \;\; A_{2\mathrm{m}(0)}/A_{3\mathrm{m}(0)} \, ; \;\; ... \, ; \;\; A_{2\mathrm{m}(n)}/A_{3\mathrm{m}(n)} \, . \tag{8}$$

In accordance with the indicated theorem, in choosing the models the similarity conditions must be fulfilled and the parameters of the full-scale object must be extended to the models. In this case, it is not necessary that the models have the same physical nature. Consequently, we can write

$$A_{\mathrm{r}} = a \, \frac{B_{\mathrm{r}}}{b} \, , \tag{9}$$

which proves the statement.

The above-considered example of modeling was used in practice for determining mechanical characteristics in examining portions of gas lines subjected to accidental deformations. For this purpose, the data presented in Fig. 3 (these data and data on conditional yield stresses have been published in [16]) have been obtained in advance under laboratory conditions on standard plane samples which were made of materials corresponding to the grades of the steels of the tubes used. The damaged portions of the tube of a gas line were locally magnetized at different pressures in the gas line and the remanent magnetic field was measured with an IN-1 stress indicator (the principle of operation of one modification of the device is presented in [17]). By employing the solution of the Lamé problem on stresses in a tube [18], we determined their values. Knowing the stresses and the values of the remanence, we determined the relative tension of the deformed portion of the tube and the conditional yield stresses using the graphs (Fig. 3). Using these results, we developed recommendations on elimination of damage.

We consider the possibility of determining the results of the phenomenon described by the quantities $C(c_1, c_2, ..., c_{i-1}, c_i, c_{i+1}, ..., c_j)$ from the phenomenon characterized by the quantities $B(b_1, b_2, ..., b_q)$. Let there be no common parameters among $b_1, b_2, ..., b_q$ and $c_1, c_2, ..., c_j$ (the case of existence of common parameters has been considered above).

In all probability, the following statement does not require a proof: *for any two physical phenomena that do not have common parameters, we can always find a third phenomenon that has at least one common*

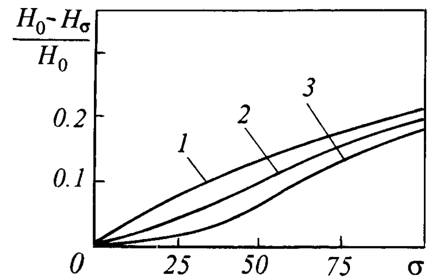

Fig. 3. Quantity $(H_0 - H_\sigma)/H_0$ versus stresses σ for deformed samples of 17G1S steel: 1) δ = 4.5%; 2) 10; 3) 13. σ, MPa.

parameter with one as well as with the other phenomenon. This statement makes it possible, if necessary, to relate any number of phenomena to the phenomenon under study.

We assume that the phenomenon having common parameters with one as well as the other phenomenon under consideration is a phenomenon characterized by the quantities $A(a_1, a_2, ..., a_n)$. Let these coincident parameters be

$$a_k = b_k \tag{10}$$

and

$$a_{k-1} = c_i \,. \tag{11}$$

By employing (10), (11), and the modeling theorem we obtain

$$C_r = c \frac{A_r}{a} \frac{B_r}{b} \tag{12}$$

or

$$C_r = f_4 (B_r) \,, \tag{13}$$

where a, b, c and A_r, B_r are the quantities determined, respectively, on models and on full-scale objects and C_r is the sought quantity. Thus, choosing corresponding phenomena and models and using (12) or (13), we can find the characteristics of one phenomenon from the characteristics of another phenomenon that has no common parameters with the phenomenon under study.

As an example of practical realization of such modeling we can refer to the determination of heat energy released in deformation of a product from the values of magnetic characteristics. In this case, one should perform operations similar to those in the above-described example of determination of mechanical characteristics at the sites of damage to a gas line and employ (12) or (13).

Conclusions. Thus, methods of modeling realized according to different schemes satisfy one definition of physical investigations of this type and allow one to relate phenomena different in nature and conduct investigations on objects that have no common parameters with the objects under study.

NOTATION

A_r, A_{1m}, $A_{2m(0)}$, $A_{3m(0)}$, ..., $A_{3m(n)}$, B, B_r, C, C_r, a, a_k, a_n, b, b_k, b_q, c, and c_j, quantities of the sought property of the object or the phenomenon; i = 1, 2, 3, ...; j = 1, 2, 3, ...; k = 1, 2, 3, ...; n = 0, 1, 2, ...; q = 1, 2, 3, ...; $f(b)$, $f_1(a_1, a_2, ..., a_n)$, $f_2(b_1, b_2, ...b_q)$, $f_3(b)$, and $f_4(B_r)$, functions; c', dimensionless constant; m_1,

m_2, ..., m_q, exponents; σ, stress; H_0 and H_σ, values of the remanence in the case of magnetization of the local region of the sample without a load and under load, respectively; δ, relative elongation of the sample. Subscripts: r and m, full-scale object and model, respectively.

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